General Certificate of Education
Advanced Subsidiary Examination
January 2012

Mathematics MPC1

Unit Pure Core 1

Friday 13 January 2012 9.00 am to 10.30 am

For this paper you must have: • the blue AQA booklet of formulae and statistical tables. You must **not** use a calculator.	

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The point A has coordinates $(6, -4)$ and the point B has coordinates $(-2, 7)$.

(a) Given that the point O has coordinates $(0, 0)$, show that the length of OA is less than the length of OB. *(3 marks)*

(b) (i) Find the gradient of AB. *(2 marks)*

(ii) Find an equation of the line AB in the form $px + qy = r$, where p, q and r are integers. *(3 marks)*

(c) The point C has coordinates $(k, 0)$. The line AC is perpendicular to the line AB. Find the value of the constant k. *(3 marks)*

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2 (a) Factorise $x^2 - 4x - 12$. *(1 mark)*

(b) Sketch the graph with equation $y = x^2 - 4x - 12$, stating the values where the curve crosses the coordinate axes. *(4 marks)*

(c) (i) Express $x^2 - 4x - 12$ in the form $(x - p)^2 - q$, where p and q are positive integers. *(2 marks)*

(ii) Hence find the minimum value of $x^2 - 4x - 12$. *(1 mark)*

(d) The curve with equation $y = x^2 - 4x - 12$ is translated by the vector $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$.

Find an equation of the new curve. You need not simplify your answer. *(2 marks)*

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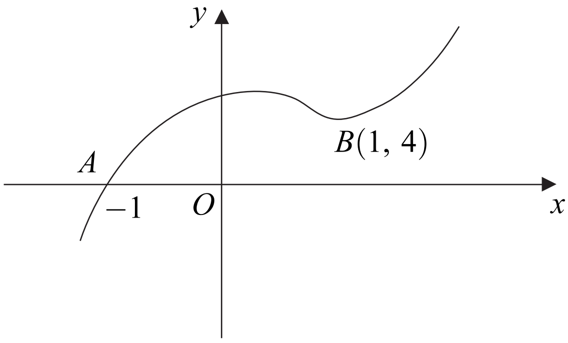
3 (a) (i) Simplify $\left(3\sqrt{2}\right)^2$. *(1 mark)*

(ii) Show that $\left(3\sqrt{2} - 1\right)^2 + \left(3 + \sqrt{2}\right)^2$ is an integer and find its value. *(4 marks)*

(b) Express $\dfrac{4\sqrt{5} - 7\sqrt{2}}{2\sqrt{5} + \sqrt{2}}$ in the form $m - \sqrt{n}$, where m and n are integers. *(4 marks)*

4 The curve with equation $y = x^5 - 3x^2 + x + 5$ is sketched below. The point O is at the origin and the curve passes through the points $A(-1, 0)$ and $B(1, 4)$.



(a) Given that $y = x^5 - 3x^2 + x + 5$, find:

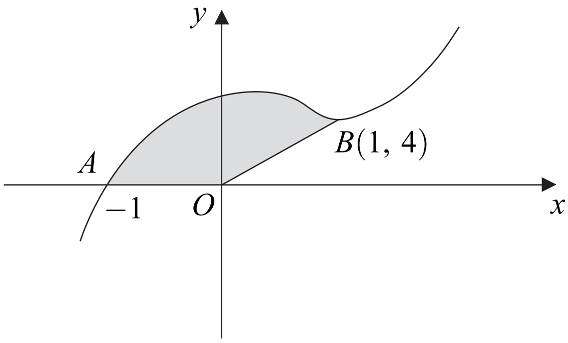
(i) $\dfrac{dy}{dx}$; *(3 marks)*

(ii) $\dfrac{d^2y}{dx^2}$. *(1 mark)*

(b) Find an equation of the tangent to the curve at the point $A(-1, 0)$. *(2 marks)*

(c) Verify that the point B, where $x = 1$, is a minimum point of the curve. *(3 marks)*

(d) The curve with equation $y = x^5 - 3x^2 + x + 5$ is sketched below. The point O is at the origin and the curve passes through the points $A(-1, 0)$ and $B(1, 4)$.



(i) Find $\displaystyle\int_{-1}^{1} (x^5 - 3x^2 + x + 5)\,dx$. *(5 marks)*

(ii) Hence find the area of the shaded region bounded by the curve between A and B and the line segments AO and OB. *(2 marks)*

Turn over ▶

5 The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 + cx^2 + dx - 12$, where c and d are constants.

(a) When $\mathrm{p}(x)$ is divided by $x + 2$, the remainder is -150.

Show that $2c - d + 65 = 0$. *(3 marks)*

(b) Given that $x - 3$ is a factor of $\mathrm{p}(x)$, find another equation involving c and d. *(2 marks)*

(c) By solving these two equations, find the value of c and the value of d. *(3 marks)*

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6 A rectangular garden is to have width x metres and length $(x + 4)$ metres.

(a) The perimeter of the garden needs to be greater than 30 metres.

Show that $2x > 11$. *(1 mark)*

(b) The area of the garden needs to be less than 96 square metres.

Show that $x^2 + 4x - 96 < 0$. *(1 mark)*

(c) Solve the inequality $x^2 + 4x - 96 < 0$. *(4 marks)*

(d) Hence determine the possible values of the width of the garden. *(1 mark)*

7 A circle with centre C has equation $x^2 + y^2 + 14x - 10y + 49 = 0$.

(a) Express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2$$

(3 marks)

(b) Write down:

(i) the coordinates of C;

(ii) the radius of the circle. *(2 marks)*

(c) Sketch the circle. *(2 marks)*

(d) A line has equation $y = kx + 6$, where k is a constant.

(i) Show that the x-coordinates of any points of intersection of the line and the circle satisfy the equation $(k^2 + 1)x^2 + 2(k + 7)x + 25 = 0$. *(2 marks)*

(ii) The equation $(k^2 + 1)x^2 + 2(k + 7)x + 25 = 0$ has equal roots. Show that

$$12k^2 - 7k - 12 = 0$$

(3 marks)

(iii) Hence find the values of k for which the line is a tangent to the circle. *(2 marks)*